# Testing relativity of simultaneity using GPS satellites

**Article** · October 2019



1 author:

Kuan Peng

# Testing relativity of simultaneity using GPS satellites

Kuan Peng 彭宽 *titang78@gmail.com*

25 October 2019

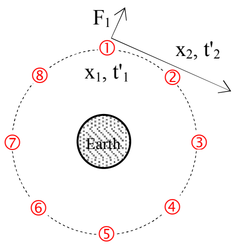
**Abstract**: Relativity of simultaneity can be measured with clocks of GPS satellites.

## 1. Time-slide in GPS system

In Special Relativity relativity of simultaneity is the fact that 2 simultaneous events occurring in a stationary frame does not appear simultaneous in a moving frame. For example, in Einstein's train thought experiment 2 simultaneous flashes of light on the platform do not appear simultaneous for the observer in the train. But relativity of simultaneity has never been tested with real simultaneous events.

For testing relativity of simultaneity we need 2 synchronized clocks moving at high speed and we will read them in a stationary frame. Fortunately, we have at hand many GPS satellites which carry precision clocks and broadcast their time, with which we can check relativity of simultaneity.

Figure 1 shows an example of 8 GPS satellites in a circular orbit. $F_1$ is the frame moving with the satellite 1, its x axis passes through the satellites 1 and 2. In a frame that coincides with $F_1$ at time 0 and stationary with respect to the Earth, the positions of the satellites 1 and 2 are $x_1$ and $x_2$. Suppose that 2 simultaneous events occur at $x_1$ and $x_2$ in the stationary frame. In the moving frame $F_1$ these same events will occur at the times $t'_1$ and $t'_2$ on the satellites 1 and 2 respectively. $t'_1$ and $t'_2$ are determined by the time equation of the Lorentz transformations which is equation (1) with v being the velocity of the satellite 1. We call $t'_2 - t'_1$ the time-slide of the event at $x_2$ with respect to that at $x_1$. The time-slide of the satellite 2 with respect to the satellites 1 is approximately expressed by equation (2) because the velocities of the satellites 1 and 2 are not parallel.




GPS satellites

Figure 1

$$t' = \frac{t - x\frac{v}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (1)$$

$$t'_2 - t'_1 \approx -\frac{\frac{v}{c^2}(x_2 - x_1)}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (2)$$

In order to get a precise expression of the time-slide of the satellite 2, let us imagine that there are n satellites between the satellites 1 and 2. In the frame of the $i^{th}$ satellite the time-slide between the satellites i and i+1 is $\Delta\tau'_i$ which is expressed by equation (3), where $\Delta\varepsilon_i$ is the distance between these 2 satellites. Equation (3) is sufficiently precise if the satellites i and i+1 are so close that their velocities can be taken as parallel. By summing all $\Delta\tau_i$ from i=1 to n, we obtain $t'_2 - t'_1$ the time-slides of the satellite 2 which is expressed in equation (4).

$$\Delta\tau'_i = -\frac{\frac{v}{c^2}\Delta\varepsilon_i}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (3)$$

$$t'_2 - t'_1 = \sum_1^n \Delta\tau'_i = -\frac{\frac{v}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}\sum_1^n \Delta\varepsilon_i \qquad (4)$$

When n is very big, the sum of all the distance $\Delta\varepsilon_i$ equals $p_2$, the length of the arc between the satellites 1 and 2, see Figure 2. So, the time-slide of the satellite 2 is precisely expressed with $p_2$ in equation (5).

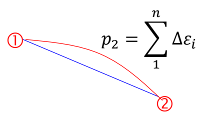



Figure 2

$$t'_2 - t'_1 = -\frac{\frac{v}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}p_2 \qquad (5)$$

For all the satellites in Figure 1, the time-slide of the $j^{th}$ satellite with respect to the satellites 1 is $t'_j - t'_1$ and is expressed by equation (6) with $p_j$ being the arc from the first satellite to the $j^{th}$ satellite and j being any number from 1 to 9. The $9^{th}$ satellite is in fact the first satellite because the orbit is a circle.

$$t'_j - t'_1 = -\frac{\frac{v}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}p_j \qquad (6)$$

The time-slide of the $9^{th}$ satellite is $t'_9 - t'_1$ and is non-zero. So, due to relativity of simultaneity the time of the $9^{th}$ satellite is different from that of the first satellite. But this is impossible in real world because the $9^{th}$ satellite is the first satellite.

On the other hand, the synchronization of the satellites is not obvious.

According to the Wikipedia page Basic concept of GPS: "The satellites carry very stable atomic clocks that are synchronized with one another and with the ground clocks." So, the satellites 1 and 2 for example must be synchronized with one clock on Earth, that is, the event "time of the satellite 1 is $t_0$" and the event "time of the satellite 2 is $t_0$" occur simultaneously on Earth. But, as the satellite 2 has a non-zero time-slide with respect to the satellite 1, the satellite 2 cannot be synchronized with the satellite 1 in orbit. Conversely, if the satellite 2 is synchronized with the satellite 1 in orbit, the reading of their time on Earth would be different. So, due to relativity of simultaneity the satellites 1 and 2 cannot be synchronized with the clock on Earth and with one another at the same time.

Nevertheless, let us compute the value of $t'_9 - t'_1$. The radius of the GPS orbit is 26 600 km (see the Wikipedia page Structure of the orbit of GPS satellites), the circumference of this orbit is 167 133 km, that is, $p_9$ = 167 133 km. The velocity of the satellites is 3.87 km/s, the speed of light is 299792 km/s. Then, the time-slide of the 9th satellite is $t'_9 - t'_1 = -7204$ ns. If this time-slide really exists but is not correctly dealt with, the GPS system would give wrong coordinates on Earth.

The coordinates computed by GPS devices on Earth using the time of satellites are actually correct, which proves that the clocks of the satellites are really synchronized with the clock on Earth and also with one another. This is impossible if relativity of simultaneity affects GPS satellites.

## 2. Time-slide and length contraction

What is the consequence if relativity of simultaneity were not true? Relativity of simultaneity is given by the time equations of the Lorentz transformations which are derived from the space equations that are the equations (7) and (8). By substituting equation (7) for x' in equation (8), we obtain equation (9) which we transform into equation (10). By substituting equation (8) for x in equation (7), we obtain equation (11) which we transform into equation (12). Equations (10) and (12) are the 2 time equations.

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad (7)$$

$$x = \frac{x' + vt'}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad (8)$$

So, the system of the 2 time equations is a linear rearrangement of the system of the 2 space equations and these 2 systems are equivalent. This is why the length-contraction-caused Ladder paradox can be explained using relativity of simultaneity. But also, a contradiction with relativity of simultaneity leads to a contradiction with length contraction.

$$x\sqrt{1 - \frac{v^2}{c^2}} = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} + vt' \qquad (9) \qquad t' = \frac{t - x\frac{v}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad (10)$$

$$x'\sqrt{1 - \frac{v^2}{c^2}} = \frac{x' + vt'}{\sqrt{1 - \frac{v^2}{c^2}}} - vt \qquad (11) \qquad t = \frac{t' + x'\frac{v}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad (12)$$

## 3. Orbital length contraction

Does the said contradiction with length contraction exist? Let us show it with geostationary satellites which are particularly appropriate for this purpose because they are stationary with respect to observers on Earth, they all move at the same velocity and their orbit is circular.

A satellite is located with respect to the center of the Earth by a vector called position vector which we can precisely determine with radars on Earth. Once the position vectors of 2 satellites are determined, we can derive the distance between them and check if this distance verifies length contraction.

In the example shown in Figure 3, the 8 geostationary satellites are immobile in the sky for an observer on Earth, that is, they are immobile in the frame of the observer which we denote by A. In a frame in space which does not rotate with the Earth, these satellites move around the Earth. This frame is denoted by B.

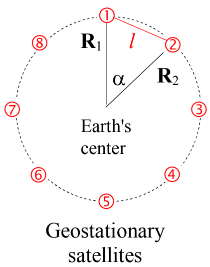


Figure 3

The position vectors of the satellites 1 and 2 are $\mathbf{R}_1$ and $\mathbf{R}_2$ in the frame A at time 0. The angle made by the vectors $\mathbf{R}_1$ and $\mathbf{R}_2$ is $\alpha$, which is the angular position of the satellite 2 with respect to the satellite 1. For n satellites that are equally spaced in orbit, the angle from one satellite to the next is always $\alpha$ and the angular position of the $i^{th}$ satellite is the angle $(i\text{-}1)\cdot\alpha$ in the frame A.

The frame B is such that the position vectors of the satellite 1 in the frame A and B coincides at time 0 and equal $\mathbf{R}_1$. In the frame B the position vector of the satellite 2 is $\mathbf{R'}_2$ and the angle made by $\mathbf{R}_1$ and $\mathbf{R'}_2$ is $\beta$. Since the angle from one satellite to the next is always $\beta$, the angular position of the $i^{th}$ satellite is the angle $(i\text{-}1)\cdot\beta$.

Let $l$ be the distance between the satellites 1 and 2 in the frame A. In the frame B the satellites being moving, the length $l$ undergoes length contraction and the distance between these 2 satellites is $l'$. The ratio of length contraction is $l'/l$, which is given by equation (13). Because $l' < l$, $\mathbf{R'}_2$ is slightly closer to $\mathbf{R}_1$ than $\mathbf{R}_2$ and we have $\beta<\alpha$.

$$\frac{l'}{l} = \sqrt{1 - \frac{v^2}{c^2}} \approx 1 - \frac{1}{2}\frac{v^2}{c^2} \quad (13)$$

The angular position of the n+1$^{th}$ satellite is $n\cdot\alpha=2\pi$ in the frame A and $n\cdot\beta$ in the frame B. $n\cdot\beta$ is smaller than $2\pi$ because $\beta<\alpha$. But, the geostationary orbit being a circle, the n+1$^{th}$ satellite is the first satellite and it is impossible that the n+1$^{th}$ satellite is not at the angular position $2\pi$ while being the first satellite.

So, length contraction in orbit creates a gap between the n+1$^{th}$ satellite and the first satellite in the frame B, which is the same type of contradiction than the contradiction with relativity of simultaneity.

The value of this gap is computed by equation (14) using the parameters of the geostationary orbit: the orbital speed is 3.0746 km/s, the radius of the orbit is 42 164 km and the circumference of the orbit is $nl$ = 264 924 km. The value of the gap is then 14 mm, which is too small to be measured. But for particles traveling in a circular accelerator at a fraction of the speed of light, this gap is significantly big.

$$n(l - l') \approx \frac{nl}{2}\frac{v^2}{c^2} \quad (14)$$

## 4. Circular accelerator

In « How to test length contraction by experiment? », I have proposed to test length contraction using n fast moving electrons in a circular accelerator. The electrons are equally spaced in the accelerator tube as Figure 4 shows. For an observer situated at the center of the accelerator and rotating with the electrons, the electrons are immobile and the length of the chain of electrons from number 1 to number n+1 equals the circumference of the accelerator tube, the n+1$^{th}$ electron being the first electron.

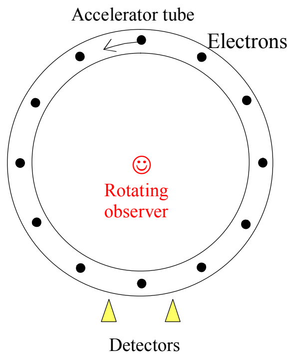


Figure 4

At the velocity of 0.865 c, the ratio of length contraction equals 0.5, which implies that, for an observer who measures the moving electrons using the detectors in the laboratory, the length of the chain of electrons equals half the circumference of the accelerator tube and he would see all the n electrons squeezed into one half of the accelerator tube and the other half is empty, as shown in Figure 5. It is impossible that the n+1$^{th}$ electron is not at the place of the first electron because they are the same electron.

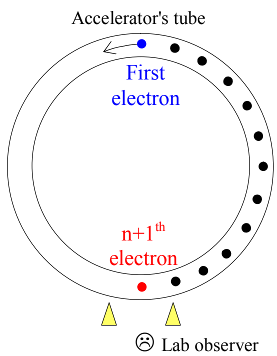


Figure 5

## 5. Comments

In section 1, I have shown that GPS satellites cannot be synchronized in accordance with relativity of simultaneity. In « Astrophysical jet and length contraction » I have shown that relativistic jets ejected by black holes do not appear denser in the fast zone than in the slow zone because the fast zone is not brighter, see Figure 6 which is the image of the relativistic jet of the active galaxy 3C 348 / Hercules A. The image of the relativistic jets from the galaxy Cygnus A in Figure 7 and that of the Quasar 3C175 in Figure 8 do not show denser fast zone either. These images indicate that the fast moving materials are not length contracted with respect to slowly moving materials.

So, we have 2 direct evidences against length contraction, relativistic jets and the working of GPS system. Sections 3 and 4 show that in theory, it is impossible that satellites in the geostationary orbit and electrons in a circular accelerator undergo length contraction.

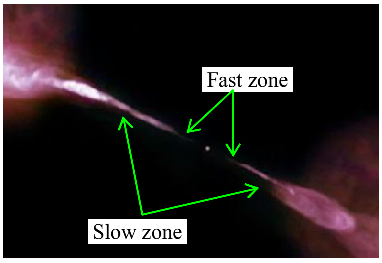


Figure 6

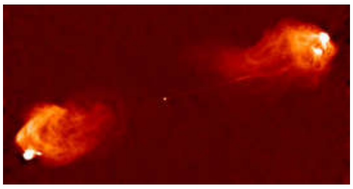


Figure 7

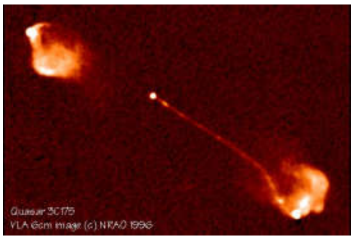


Figure 8