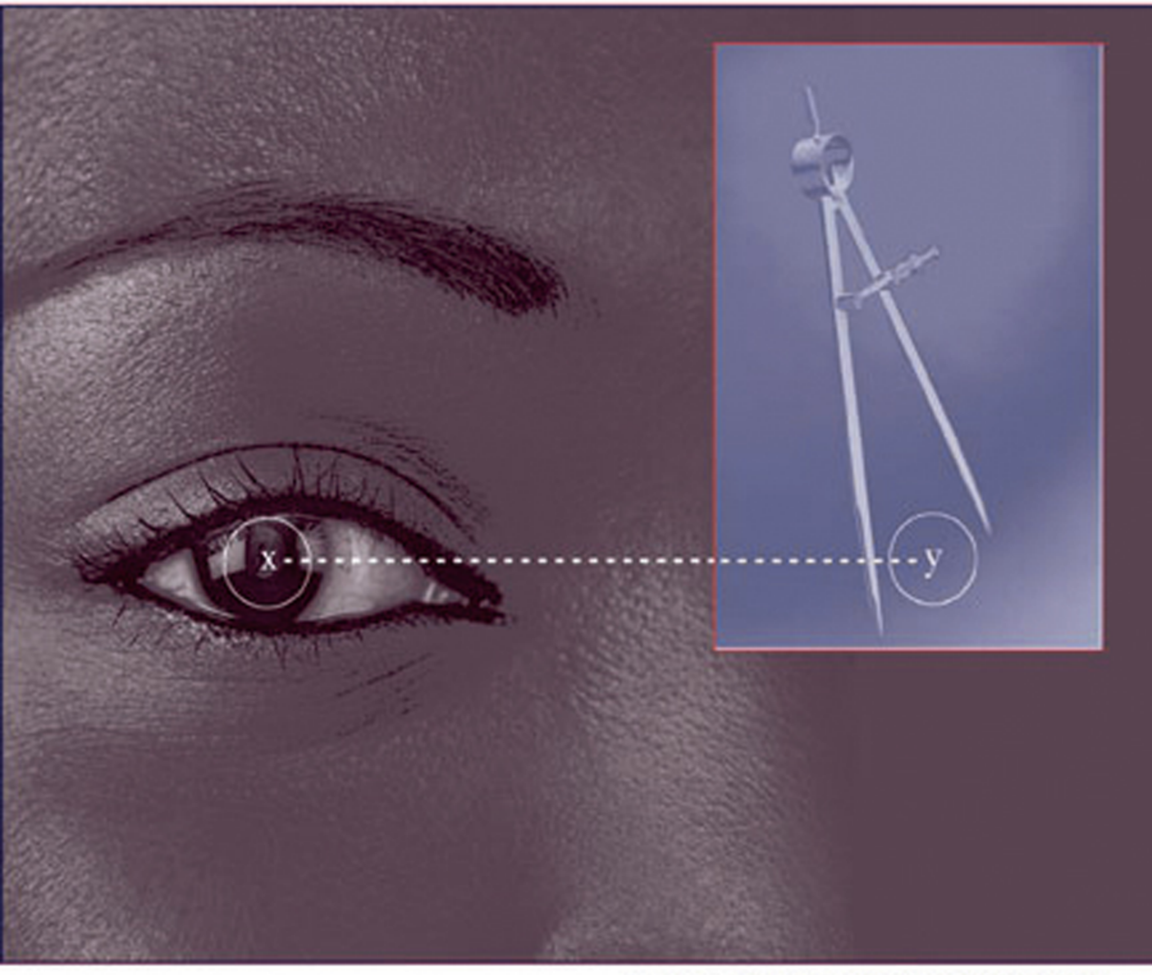
THE GEOMETRIES OF VISUAL SPACE



MARK WAGNER

The Geometries
of Visual Space

The Geometries of Visual Space

Mark Wagner
Wagner College

LAWRENCE ERLBAUM ASSOCIATES, PUBLISHERS
2006 Mahwah, New Jersey London

Camera ready copy for this book was provided by the author.

Lawrence Erlbaum Associates, Inc., Publishers
10 Industrial Avenue
Mahwah, New Jersey 07430
www.erlbaum.com

Cover design by Kathryn Houghtaling Lacey

Library of Congress Cataloging-in-Publication Data

Wagner, Mark.
The geometries of visual space / Mark Wagner.
p. cm.

Includes bibliographical references and index.

ISBN 0-8058-5252-2(cloth : alk. paper)
ISBN 0-8058-5253-0 (pbk. : alk. paper)
1. Space perception. 2. Visual perception. I. Title.
BF469.W35 2005
152.14′2—dc22 2005051008
CIP

Books published by Lawrence Erlbaum Associates are printed on acid-free paper, and their bindings are chosen for strength and durability.

Printed in the United States of America
10 9 8 7 6 5 4 3 2 1

This book is dedicated to John C. Baird

—mentor, collaborator, friend, and psychophysical wise man.

It is also dedicated to Susan Bernardo and Katie MacDonald,

for their love and support over the years.

Contents

Preface

When most people think of space, they think of physical space. *Physical space*, which is primarily the concern of physicists and geometers, is defined in reference to objective, physical measures such as rulers and protractors. As a perceptual psychologist, I am interested in another sort of space: Visual space. *Visual space* concerns space as we consciously experience it, and it is studied through subjective measures, such as asking people to use numbers to estimate perceived distances, areas, angles, or volumes.

Space perception is an important area to consider for a number of reasons. First, the study of space perception has a long pedigree. Many of the greatest philosophers and scientists in history including Descartes, Reid, Berkeley, Hume, and Kant have examined how well our perceptions of space match physical reality. The space perception problem has concerned some of the greatest minds in the history of psychology as well, including Helmholtz, Luneburg, Titchener, Wundt, James, and Gibson. Space, together with time, is the fundamental basis of all sensible experience. Understanding the nature of our spatial experience, then, addresses one of the most basic intellectual problems. Second, psychology began as the study of conscious experience. Behaviorism arose in the 1920s by asserting the proposition that it is impossible to say anything significant about conscious experience. Behaviorism is just part of a larger materialist philosophy that pervades modern science and medicine. I believe this materialist philosophy is over emphasized, and that consciousness is at least as fundamental and important as the physical world. This work on space perception is an attempt to show that one can develop a sophisticated and coherent understanding of conscious experience. Finally, there are potential practical applications of work on this topic. In the real world, predictable errors in spatial perception can have very real consequences, from landing planes badly to driving mistakes that can cost lives.

Numerous studies have found that physical space and visual space can be very different from each other. This past work has demonstrated that mismatches between physical and visual space are not isolated occurrences, but that large, systematic mismatches regularly occur under ordinary circumstances. This book reviews work that explores this mismatch between perception and physical reality. In addition, this book describes the many factors that influence our perception of space including the meaning we assign to geometric concepts like distance, the judgment method we use to report our experience, the presence or absence of cues to depth, the orientation of a stimulus with respect to our point of view, and many other factors.

Previous theorists have often tried to test whether visual space is best described by a small set of traditional geometries, such as the Euclidean geometry most of us studied in High School or the hyperbolic and spherical geometries introduced by 19th-century mathematicians. This "synthetic" approach to defining visual space relies on laying out a set of axioms characteristic of a geometry and testing the applicability of the axioms. This book describes this sort of research and demonstrates that the synthetic approach has largely failed because the empirical research commonly does not support the postulates or axioms these geometries assume.

I take a different approach based on what mathematicians call *metric functions*; that is, I attempt to specify the measurable properties of visual space, such as distances, angles, and areas, using functions that take into account the location of a stimulus in physical space and other psychological factors. The main theme of this book is that no single geometry describes visual space, but that the geometry of visual space depends on stimulus conditions and mental shifts in the subjective meaning of size and distance. Yet, despite this variation, our perceptions are predictable based on a set of relatively simple mathematical models.

Although this work is primarily intended for scholars in perception, mathematical psychology, and psychophysics, I have done my best to make this discussion accessible to a wider audience. For example, chapter 2 reviews the mathematical, philosophical, and psychophysical tools on which this book relies at what I believe is a very readable level. Because of this, I believe this book would also make for a good graduate-level textbook on space perception.

Plan of the book. The first two chapters contain philosophical, mathematical, and psychophysical background material. Visuals space is defined, and I explain why the problem is important to study. These chapters trace the history of philosophical work on space perception, which antedates psychology. They also explain how mathematicians approach geometry, describe some of the most important and widely known geometries, and discuss the psychophysical techniques used to explore visual space.

Chapter 3 looks at synthetic approaches to space perception including work on hyperbolic, spherical, and Euclidean geometries. I lay out the axioms for geometries of constant curvature and consider the extent to which these axioms are supported by empirical work. Chapter 4 proposes an alternative way to investigate the geometry of visual space, the analytic approach. Here, geometries are defined by using coordinate equations to express the metric properties of the space, such as distance, angle, area, and volume. I describe ways of assigning coordinates to visual space, talk about the origin of visual space —the egocenter, and talk about the general form of equations to describe metrics. Finally, I demonstrate that visual space violates the assumptions of one of the most general types of geometries, metric spaces.

The next three chapters review the three other major domains of psychophysical research on space perception. Chapter 5 presents a meta-analysis of studies that ask observers to directly estimate size, distance, area, angle, and volume. This meta-analysis examines how judgments of the measurable properties of visual space depend on contextual factors such as instructions, cue conditions, memory vs. direct judgment, the range of stimuli, judgment method, and so on. Chapter 6 looks at the size constancy literature in which observers are asked to adjust a comparison stimulus to match a variety of standards at different distances away. This chapter discusses the history of this literature and considers the effects of many variables on size constancy judgments such as instructions, cue conditions, age, and stimulus orientation. Chapter 7 discusses research that takes a multi-dimensional approach toward studying visual space. These studies look at how size and angle judgments change when stimuli are oriented horizontally, vertically, or in-depth. In all three chapters, mathematical models are presented that integrate data presented in the literature reviews.

Chapter 8 talks about how spatial experience is influenced by memory. In particular, I review factors that affect the development and structure of cognitive maps, including individual difference variables such as age, navigational experience, gender, and personality. In addition, it describes the types judgment errors that are unique to cognitive maps. Chapter 9 summarizes and synthesizes the data and theories discussed in the earlier chapters of the book. In addition, this chapter discusses spatial experience arising from modalities other than vision.

Acknowledgments. This book would not have been possible without the assistance of many people. First of all, I would like to thank my editors at Lawrence Erlbaum Associates, Bill Webber and Lori Stone, who helped guide me through the publication process in a gentle and professional way. In addition, I wish to thank Nadine Simms for her help with the production process. I'd also like to thank Jim Brace-Thompson of Sage Publications for the encouragement and help he provided shortly after I began writing this book.

I am also grateful to Elaina Shapiro, my undergraduate assistant, who helped me track down many hundreds of the articles that have gone into this work. She always approached this monumental task with good humor and enthusiasm. In addition, I would like to express my appreciation to Evan Feldman and Heather Kartzinel, whose research has contributed to this book.

I'd like to thank Stephen W. Link and several anonymous reviewers who gave me sage advice that helped improve the quality of the final product. Steve, in particular, gave me incredibly detailed feedback and much needed encouragement. Thanks, Steve!

I'd also like to acknowledge Susan Bernardo and Katie MacDonald for their love, tolerance, and support. Finally, I'd like to thank John C. Baird, my mentor, collaborator, and friend. This book would not have been possible without the vision and professionalism he displayed throughout our many collaborations.

—Mark Wagner

1

Introduction
Contrasting Visual, Experiential, and Physical Space

> Time and space are the two pure forms of all sensible perception, and as such they make *a priori* synthetic propositions possible.
> Immanuel Kant, *Critique of Pure Reason*

Contrasting Conceptions of Space

This book will investigate the properties of our visual perceptions of space. The concept of space has been an object of speculation and dispute throughout the history of philosophy and science. Great philosophers and scientists—Immanuel Kant, Thomas Reid, Henri Poincaré, Issac Newton, and Albert Einstein (to name a few)—have considered space (together with time) to be one of the cornerstones on which existence is based and from which philosophy and science arise.

At the outset, two terms need to be defined and distinguished: *physical space* and *visual space*. While it is tempting to distinguish between the two by saying that the latter reflects conscious experience and the former does not, I believe one must resist this temptation. Both concepts reflect aspects of our experience of the world. But the attitude we take toward that experience differs between physical and visual space. Perhaps, I risk offending some readers by reviving the shadows of Wundt and Titchener, but in perception at least, Titchener was accurate in observing:

> All human knowledge is derived from human experience; there is no other source. But human experience, as we have seen may be considered from different points of view.... First, we will regard experience as altogether independent of any particular person; we will assume that it goes

> on whether or not anyone is there to have it. Secondly, we will regard experience as altogether dependent on the particular person; we will assume that it goes on only when someone is there to share it. (Titchener, 1900/1909, p. 6)

However, conscious experience can be a rather slippery conceptual fish to grapple with. A scientist can only theorize based on solid data, and consciousness does not itself make marks on paper nor does it directly leave other physical traces that can be studied at leisure.

Harvey Carr once noted, “Consciousness is an abstraction that has no more independent existence than the grin of a Cheshire cat” (Carr, 1925). To open our experience to scientific investigation, we must rely on objectively observable behaviors and verbal reports that attempt to capture some aspect of experience, and we must resort to operational definitions of our concepts in order to render them concrete enough to use.

With this in mind, let me define my terms. By physical space I mean the space revealed to us by measuring devices such as rulers and protractors. Physical space is objectively defined; that is, the properties of physical space are largely observer independent. By visual space, I mean the space revealed by the psychophysical judgments of an observer. Visual space is not objectively defined; that is, the properties of visual space may depend critically on certain aspects of the observer, such as location in physical space, experimental conditions, and the mindset of the observer.

Defining visual space this way sidesteps the central issue: do the judgments people give accurately reflect their subjective experience of the world? Are the introspective reports that people generate a fair reflection of what is really witnessed internally? No doubt I would be wisest to simply drop the issue; however, I am too much of a philosopher to pass on without venturing an opinion.

Let me boldly state my own equivocal belief. While I believe that observers do attempt to base their judgments on their subjective experience of the world and I believe they really do try to be accurate, it is impossible to say how well they accomplish their goal. It is impossible to independently verify what is really in the subjective experience of an observer. The closest proxies we have are the judgments themselves.

Of course, if we did not believe that the numbers generated in psychophysical experiments reflected something of a person’s internal experience, we would quickly lose interest in the subject. Why would one really care about mere number generating responses? A true behaviorist should find perception boring.

Geometry and space. A variety of geometries have been employed to describe physical space at different levels of scale. When the distances under consideration are large, Einstein (1922) pointed out that a hyperbolic geometry might best describe physical space. When the slightly less grandiose distances of the earth's surface are considered, a spherical (or elliptical) geometry makes sailing or flying around the world quicker and more efficient. Yet, if we confine ourselves to that range of distances which humans commonly experience; that is, if we confine ourselves to the ecological level of analysis mentioned by Gibson (1979); then any curvature in the earth's surface or in the fabric of space itself is small enough to be ignored. The world is Euclidean. When distances are measured by a ruler, the square of the hypotenuse of a right triangle is equal to the sum of the squares of the two legs to a high degree of approximation—just as Euclidean geometry would predict. When angles are measured by a protractor, the sum of the angles of any triangle is always very, very, very close to 180°—just as Euclidean geometry would predict.

The same definite conclusions cannot be made regarding visual space. People are capable of thinking about geometric concepts in different ways. By a simple mental shift, we can think of the distance from home to work as the crow flies, as the length of the path to get there, as the time it takes to drive, or as a segment of the "great circle" that intersects the two points. We can think of distance as the physicist sees it or take the artist's perspective and see distance as the amount of canvas lying between two objects in a painting. One time we can use category estimation to judge distance and try to keep differences between categories subjectively identical while another time we use magnitude estimation and try to reflect the ratio of the subjective sizes of targets; and emphasizing these different mathematical aspects of the situation leads to very different psychophysical functions. Which of the geometries of visual space that result from these different perspectives is correct? I believe it is best to simply admit that no single view is correct, but that they all are. All may be valid descriptions of our varying subjective experience.

In addition, our experience is influenced by the situation we find ourselves in. Trying to judge the distance to an on-coming car is more difficult at night than it is during the day. Things that are far away can seem different than when they are brought close to us, and the angle from which we regard an object can make a difference to our perceptions of it. The world can seem large in the mind of a child, but the adult who returns to the old neighborhood is struck by how small and underwhelming things seem.

As we will see later, many have attempted to specify *the* geometry of visual space, but in my view that enterprise is hopeless from the outset. There is no single geometry that describes visual space, but there are many. The geometries of visual space vary with experience, with mental set, with conditions, and with time.

The purpose of this book is to determine how the geometry of visual space changes along with conditions. In addition, as part of that, this book will look at the changing relationship that exists between physical space and our visual perceptions of it.

Memory and space. The foregoing hints that physical and visual spaces are not the only ones of interest to the psychologist. What of memorial space, space as we remember it based on a past viewing of an object or setting? Even if one believed that space as it is directly perceived is both accurate and Euclidean as a Gibsonian would suggest, a psychologist would have good reason to suppose that the process of memory would distort our judgments into a very non-Euclidean form. Memories are incomplete and reconstructed.

Cognitive maps are another step away from direct perception. Cognitive maps refer to our mental representations of the layout of our surrounding environment. Cognitive maps generally concern large-scale environments that are too big to ever be seen at one time (except perhaps from an airplane or a space ship); so, cognitive maps are constructed across time based on our unfolding experience. As we will see later, cognitive maps are riddled with holes (that represent unexperienced territories), distortions, discontinuities, and non-spatial associations. A complete characterization of cognitive space is not only non-Euclidean; it is probably non-Riemannian. In fact, there may be no simple mathematical system that could ever fully characterize the richly chaotic nature of our cognitive maps. Cognitive maps may consist of a patchwork of loosely connected parts.

From a psychological standpoint, memorial space and cognitive maps certainly deserve our attention, and this book will describe something of their nature. Once more, the family of geometries that describe human experiences expands. Who could think there might be only one?

Experiential space. Of course, one need not stop here. A more general conception than visual space is that of experiential space. By experiential space I refer to our experience of space of any kind. By its very nature, the term visual space excludes spatial perceptions based on the other senses. Yet, clearly we do perceive space in extra-visual ways. Not only does it make sense to speak of visual space, but one may also meaningfully speak of auditory space, haptic space, gustatory space, kinesthetic space, proprioceptive space, and olfactory space. This book will largely confine itself to vision because the vast majority of research studies on spatial perception concern visual stimuli, but I will have a few words to say about these other spaces in various places in this book.

Why Is This Problem Important?

A noble intellectual pedigree. The problem of space perception is one with a long and prestigious pedigree. According to Wade (1996), ancient Greek philosophers including Aristotle and Euclid recognized that spatial perception did not always correspond to physical measures and that variables such as binocularity, aerial perspective, and distance to the stimulus can alter size estimates. Roman era thinkers including Galen, Lucretius, and Ptolemy noted that variables like linear perspective and the orientation of a stimulus can lead to breakdowns in size constancy. The great 11th Century Islamic philosopher, Ibn al-Haytham, spoke of the effects of stereopsis and familiar size on spatial perception. Leonardo da Vinci reiterated the importance of binocularity and aerial perspective on size perception.

Philosophers throughout the modern era often wrote about spatial experience as part of their systems of philosophy. Wade (1996) mentions Francis Bacon's and René DesCartes's interest in the problems of space perception. As will be discussed at length in Chapter 2, Berkeley, Hume, Kant, Reid, Poincaré, and Husserl all held well-developed views on the geometric character of our spatial experience.

Interest in the problem of space perception also played an integral role in the development of psychology as a discipline. Helmholtz (1868/1921) extensively wrote about space perception and empirically investigated the problem as part of his assault on Kantian philosophy. Weber's studies of two-point limen in touch were largely motivated by his wish to understand how humans develop our sense of space. Other early founders of psychology, including Titchener and James, wrote extensive chapters (or even multiple chapters) on space perception in their foundational works on psychology. In fact, the longest single chapter in James's two-volume *The Principles of Psychology* is dedicated to the subject. Wundt, whom some consider the founder of psychology, was so dedicated to studying the nature of space perception that James said of him: "Wundt has all his life devoted himself to the elaboration of space theory" (James, 1890, p. 276). (By the way, I tend to agree with Link (1994, 2002) that Fechner is a better candidate for the role of psychology's founder than Wundt. While Wundt may have been better at self-promotion, psychology was alive and well before he ever came on the scene.)

Harvey Carr (1935), the great American Functionalist, wrote an entire book on space perception. In addition, when G. Stanley Hall was granted the first Ph.D. ever awarded in psychology in America, his dissertation was on (you guessed it) space perception (Boring, 1950).

In short, some of the greatest philosophers and psychologists in history focused considerable attention on the problems of space perception. The present book follows this rich tradition and reconceptualizes our spatial experience in the light of the massive body of empirical research performed in more recent years.

Space is foundational. These great minds devoted so much of their attention of spatial experience for a very good reason. Space is foundational. The universe itself may represent little more than the interplay of space, time, and energy. Modern physics seeks to explain gravity, black holes, and the expansion of the universe in terms of alterations in the fabric of space.

Psychologically, space is one of the fundamental building blocks of human experience. Without a conception of space, object perception and meaningful interaction with the world would be impossible. One literally could not live without some ability to sense the layout of the world. At times, one literally cannot live when this perception is in error at a critical time.

Kant (1781/1929) firmly believed that spatial experience served as the base out of which our phenomenal experience grows. In his words:

> Space is a necessary *a priori* representation, which underlies all outer intuitions. We can never represent to ourselves the absence of space, though we can quite well think it as empty of objects. It must therefore be regarded as the condition of the possibility of appearances, and not as a determination dependent on them. (Kant, 1781/1929, p. 24)

Like Kant, I feel that spatial experience represents something particularly fundamental that deserves detailed study. Unlike Kant, I believe that explicating the nature of visual space is an empirical, rather than a logical, *a priori* issue. This book describes the nature of visual space as revealed by the research literature.

A paradigm for measuring mind. Fechner (1860) and Wundt (1874/1904) attempted to apply mathematical tools and the scientific method to the study of consciousness, and for a while all of psychology focused on the study of conscious mind. But as time passed, psychology became ever less interested in consciousness and ever more interested in behavior. Why did this happen? Some believe early Structural Psychology died due to its methodological defects. Carr (1925), who did not wholly reject the introspective method of the Structuralists, pointed out the defects of introspection. He felt that introspection was too difficult to do to give much detailed information about consciousness, that introspective reports were not subject to independent verification, and that Structuralists tended to rely on trained observers whose observations were too easily influenced by their knowledge of the research hypotheses—what James (1890) referred to as the Psychologist's Fallacy.

A more fatal line of attack on introspection came from Watson (1914, 1919, 1924, 1925). Watson felt that it was impossible to make any real progress with a science based on introspection and that the whole enterprise could be dismissed as irrelevant. "The psychology begun by Wundt has thus failed to become a science and has still more deplorably failed in contributing anything of a scientifically usable kind to human nature" (Watson, 1919, p. 3).

While I realize that modern psychology has lost much of its behaviorist character, Watson's challenge is still one I take very seriously. Is it possible to take introspective reports and develop them into an organized, sophisticated, developing body of knowledge? If Watson is right, then it is not only difficult to study the mind, but mind becomes a mere wisp or vapor of no importance.

But, one can develop a sophisticated science based on introspective reports. And I believe no area of psychology is fitter to demonstrate this point than the spatial perception literature. Space perception can be seen as a paradigm of success in the study of mind. This book is an attempt to answer Watson's charge.

More recently, a second serious charge was leveled against the whole enterprise of psychophysics. Lockhead (1992) accused psychophysicists of generating a sterile discipline that consists of a series of unidimensional investigations that fail to adequately grapple with the effects of context on judgments. I see the present book as a lengthy refutation of Lockhead's charge. When taken together the spatial perception literature paints a rich, multidimensional picture that dynamically changes as a function of contextual variables.

Practical applications of visual space perception. James (1907/1964) felt that scientists could be divided into two groups based on their temperaments. The forgoing justifications might appeal to those with what James referred to as a "tender-minded make-up," but might not convince those with a more "tough-minded make-up." A final justification for the study of space perception might even satisfy readers of the hard-nosed persuasion. Space perception research can have many practical applications.

For example, Kong, Zhang, Ding, and Huikun (1995) found that accident-prone railroad drivers had poorer spatial perception skills, particularly those related to depth perception, than safe railroad drivers. Another group of Chinese researchers divided drivers into excellent, regular, relatively poor, and accident prone groups based on driving test scores and accident records and found that the worst drivers had significantly poorer visual depth perception (Zhang, Huang, Liu, & Hou, 1995). Hiro (1997) noted that the faster people drive, the more they underestimate the distance to the car ahead of them. Given that it takes more time to stop at faster speeds, this underestimation of distance could prove to be fatal.

Another skill that drivers need is the ability to read maps accurately. Gillan, Schmidt, and Hanowski (1999) found that contextual variables such as Müller-Lyer Illusion elements in the map can lead to map reading errors.

Pilots need to perceive accurately spatial layout in order to land their planes safely. Lapa and Lemeshchenko (1982) found that pilots who use an egocentric coordinate system have slower reaction times and make more errors in judging layout than those using a geocentric coordinate system. Of course, these pilot errors can cost lives.

Other pilot tasks involving spatial perception include searching for places or landing fields, flying in formation, aerial refueling, collision avoidance, weapons targeting, and low-level flight (Harker & Jones, 1980). Westra, Simon, Collyer, and Chambers (1982) found that landing on aircraft carriers depended more on a pilot's spatial abilities and training than on equipment factors. Unfortunately, distance judgments made from up in the air often lack many of the cues to depth usually found for terrestrial observers. Roscoe (1979) found that spatial perception was particularly difficult at dusk or in the dark, when flying over water, and when coming out of a bank of clouds. Roscoe (1982, 1985) also found that inaccuracies in spatial perception occur when pilots accommodate to their dark focus depth or on the cockpit window rather than on objects external to the cockpit.

If pilots have difficulty judging spatial layout because cues to depth are often absent, astronauts are likely to experience even more difficulty in determining the location of objects external to their capsule since many cues to depth are totally absent in space. Understanding spatial perception in outer space can be an important area of future research to assist the development of projects such as the International Space Station.

Space perception can also be critical for sports performance. For example, Issacs (1981) found that poor depth perception was an important variable in free-throw shooting in basketball. Similarly, McBeath, Shaffer, and Kaiser (1995) and Shaffer, Karauchunas, Eddy, and McBeath (2004) have shown how complicated the simple process of catching a baseball can be. In fact, Oudejans, Michaels, Bakker, and Dolne (1996) indicate that stationary observers are very poor at judging the catchableness of a baseball compared to moving observers. Obviously, this suggests that a running start may be an essential trick to being a good outfielder.

Another place where the ability to accurately perceive spatial layout is important is in surgery. Reinhardt and Anthony (1996) found the ability to engage in remote operation procedures involving internal cameras depended on the adequacy of depth and distance information. Conflicts between monocular and stereoscopic cues proved particularly problematic.

In another recent medical study, Turano and Schuchard (1991) found spatial perception deficits often result from macular and extramacular-peripheral visual field loss. (Although some subjects with quite extensive loss showed normal space perception.) What is more, these perceptual deficits occurred even outside of the damaged areas and when visual acuity is good.

Inaccuracy in distance estimates can also be an issue in some court cases. At least as far back as Moore (1907) legal scholars have been aware of a multitude of variables that negatively affect the reliability of witness testimony regarding spatial layout and the speed of movement. These variables include the amount of time witnesses observe a layout, the passage of time since the incident, the emotional state of the witness, motion in the object observed, darkness, and whether the incident is seen through water or air. At times witness estimates of layout can be critical information in courtroom testimony.

Because our ability to correctly perceive spatial layout is necessary for proper performance in so many areas, it is important to know which factors lead to spatial estimation errors so that we may engage in actions that may eliminate those errors. This book will examine many of these factors.

The Plan of the Book

All of the chapters of this book are directed at two central purposes: to describe our perceptions of visual space and to compare these perceptions to physical layout. The domain delimited by these objectives still covers a vast amount of material, because these two problems have many facets and can be approached from many different directions. The remainder of this chapter describes the various approaches this book takes toward addressing these central objectives. It lays out the basic plan of the book, briefly describing the contents of each of the chapters that follow.

Chapter 2. Like all issues in psychology, the questions discussed in this book arise within a larger historical context. As someone who has a deep interest in the history of psychology—I even co-edited a book on American Functionalism (Owens & Wagner, 1992)—I believe it is important to set up the discussion that follows by providing a bit of this historical background. Chapter 2 also describes some of the mathematical and psychophysical tools that may be used to characterize the geometries of visual space.

In particular, this chapter first discusses the ways mathematicians addressed geometry across history. Secondly, like the rest of psychology, the study of visual space grew out an attempt to apply scientific methods to a long-standing philosophical problem. This chapter speaks about early philosophical approaches to space perception. Finally, Chapter 2 discusses how the study of space perception fits into the wider domain of psychophysics, which provides the basic techniques necessary to paint a picture of visual space.

Chapter 3. Mathematicians define a space in two ways: synthetically and analytically. In the synthetic approach to geometry, the mathematician lays out a set of postulates that define a geometry and deduces theoretical statements that are the consequence of these statements. Chapter 3 describes the work of psychologists who applied this synthetic approach to visual space, particularly emphasizing Luneburg's hyperbolic model. Theoretical works proposing Euclidean and other more exotic geometries are also mentioned. Chapter 3 discusses the assumptions made by these theorists, the predictions made by each theory, and the degree to which empirical research supports these synthetic models.

Chapter 4. My approach to describing visual space is analytic. In an analytic geometry, a set of coordinates is assigned to a space and equations are used to describe the measurable properties of the space like distance, angles, and area. This chapter discusses the advantages of the analytic approach. It describes methods for assigning coordinates to visual space, the location of the origin of visual space, and general formulas for distance, area, and volume judgments. The chapter also talks about how visual space sometimes fails to satisfy the axioms of a metric space, one of the most general forms of an analytic geometry, and describes some dramatic consequences of this failure.

Chapter 5. An expansive literature shows that judgments of the measurable properties of visual space depend on contextual factors such as instructions, cue conditions, memory vs. direct judgment, the range of stimuli, judgment method, etc. Chapter 5 performs a meta-analysis of the effects of these factors on the parameters of psychophysical functions and on the goodness of fit of these psychophysical functions. This meta-analysis is based on over seven times as many studies and experimental conditions than any previously published meta-analysis on space perception. Multiple regression analyses of this data produce a set of general psychophysical equations for distance, area, and volume judgments as a function of contextual conditions. Angle judgments are also briefly examined.

Chapter 6. A second spatial perception literature concerns the perception of size constancy. In this literature, a near comparison is adjusted to match the size of standard stimuli at varying distances from the observer, at varying orientations, and under varying cue conditions. This chapter discusses the history of this literature and considers the effects of many variables on size constancy judgments. It also develops a theory to explain the results that is a generalization of the classic Size-Distance-Invariance Hypothesis. The virtue of the present theory is that it allows one to unify the size constancy literature and the psychophysical literature addressed in the previous chapter. Finally, the chapter briefly talks about the link between size-constancy and the moon illusion.

Chapter 7. The vast majority of the psychophysical literature is based on unidimensional judgments, where depth and egocentric distance are looked at independently from frontal size perception. Chapter 7 talks about a few exceptions to this unidimensional rule that look at spatial judgments as a function of two or even three dimensions simultaneously. I describe two of my own studies of this type and present several models to describe this data. Here, at last, we create models that fully specify the geometry of visual space under a given set of conditions. The rest of the chapter discusses other work of this type. These studies look at changes in visual space as a function of distance from the observer, elevation of gaze, and the presence of context-defining objects. The chapter also mentions evidence for the presence of multiple visual systems, one that guides motion and the other that produces visual experience.

Chapter 8. Memory adds yet another layer of complexity to the analysis of spatial experience. This chapter contrasts the data and theoretical approaches produced by the direct perception and memory literatures, particularly focusing on the cognitive mapping literature. It describes the structural elements of cognitive maps and how cognitive maps are acquired across time. I look at the affect of individual difference factors on cognitive maps such as age, navigational experience, gender, and personality. I also look at the nature of the errors that cognitive maps contain. After this, Chapter 8 compares the psychophysical judgments of size and distance that observers give under direct perception, memory, and cognitive mapping conditions. The chapter also develops a theoretical framework for understanding these data. Finally, I list a few objections to the cognitive-science paradigm that pervades much of this literature and mention an alternative way to think about memory.

Chapter 9. The final chapter summarizes and synthesizes the data and theories discussed in the earlier chapters of the book. In addition, Chapter 9 will touch on spatial experience arising from modalities other than vision. Finally, the chapter discusses the ecological, philosophical, and practical implications of the spatial perception literature.

The End of the Beginning

In summary, a wealth of data that are discussed in the following chapters indicates that visual space is different from physical space. In fact, the geometry that best describes visual space changes as a function of experimental conditions, stimulus layout, observer attitude, and the passage of time.

In addition, the problem of human spatial perception is one of great antiquity, long-standing philosophical import, and considerable practical significance. The spatial perception literature is well enough developed to convincingly show that a sophisticated science can be based on the introspective reports of observers.

2

Traditional Views of Geometry and Vision

Like most psychological problems, the problem of space perception exists within a context larger than itself. This chapter provides a bit of this context. In particular, this chapter examines the historical background of the problem and looks at the empirical and analytical tools available to describe visual space.

The first leg of this contextual tour looks at the approaches mathematicians use to define the geometry of a space. Following this, we discuss the works of early philosophers whose views about visual space naturally led to more recent psychological developments. Finally, this chapter briefly discusses the psychophysical methods that are employed to empirically measure visual space perception.

Geometry as the Mathematician Sees It

Because the problem of visual space perception is explicitly geometric in nature, a logical place to begin searching for tools to work on the problem is with geometry. How do mathematicians define a space? According to Kline (1972), there are two general approaches to geometry. One is synthetic, and the other is analytic.

Synthetic approaches. Ancient Babylonia and Egypt possessed forms of geometry; however, these geometries were concrete, primitive, and lacked unifying principles. This early work focused on solving practical problems associated with flood control, building, and trade. They relied on approximation rather than exact numbers. For example, π was thought to be three. While these ancient mathematicians anticipated many important elements of geometry (such as the Pythagorean Theorem), their works were empirically derived. They lacked the modern concept of proof, and the various mathematical findings were not integrated into a coherent structure.

The first real sophistication in mathematics began with the classical Greeks, who created many geometry theorems. The earliest proof is generally attributed to Thales about 600 BC. Over the next few centuries, the Pythagoreans and others added many new geometrical proofs. Euclid theorems organized these theorems into a coherent structure in his book *Elements*.

In this book, Euclid laid out proofs for 465 geometrical propositions. Euclid's method, however, was of far greater importance than this impressive number of proofs. Euclid began his development by making a list of definitions and postulates. His ten postulates consisted of global, rather than algebraic, assumptions. The most famous example is the Parallel Postulate that states "Through a point P not on a line L, one and only one parallel to L can be drawn." Euclid (and others subsequently) then deduced his many theorems based on these definitions and postulates. Such a geometry, consisting of global definitions, postulates, and theorems, is called a synthetic geometry.

For over a millennium, mathematicians believed that Euclid's geometry was the only one possible. Asking what geometry best describes visual space would have made no sense to them. Visual space could only be Euclidean.

The self-evident certainty of Euclidean geometry crumbled in the early 19th century as a result of mathematical investigations of the Parallel Postulate. Euclid's Parallel Postulate had always been unsatisfactory to mathematicians. In 1733, the Jesuit mathematician Saccheri vainly attempted to prove the Parallel Postulate based on the other nine postulates. While other mathematicians largely rejected the "proof" he generated, his work induced others to take an interest in the problem.

Finally, in 1829, Lobatchevsky demonstrated not only that the Parallel Postulate could not be proved but that a perfectly consistent geometry could be constructed from the assumption that more than one parallel exists to a line through a point not on the line. A few years later Bolyai (1833) published his work demonstrating the same point. (Gauss's notes indicated that he had developed similar proofs earlier than Lobatchevsky and Bolyai, but he never published the work).

The geometry defined by this new form of parallel postulate is called a hyperbolic geometry. A hyperbolic geometry has many properties that are different from those in Euclidean geometry. In a hyperbolic geometry, the sum of the angles of a triangle is less than 180°. The "straight" lines of the space are shaped like hyperbolas. No infinity exists in the hyperbolic space; that is, the space is bounded.

In 1854, Riemann invented a third type of synthetic geometry that arises from another variant of the Parallel Postulate. In this case, Riemann assumed that no parallels to a line could be drawn through a point not on the line. Such a geometry is called a spherical geometry. A simple example of a spherical geometry is the surface of the earth. Here, "straight lines" are circles whose centers are coincident with the center of the earth. All lines (known as Great Circles) defined this way must intersect at two points.

A spherical geometry has a number of other interesting characteristics. First of all, all lines have a finite length. Because pairs of lines intersect at two points, spherical geometry violates Euclid's postulate that two straight lines cannot enclose a space. The sum of the angles of a triangle is always greater than 180° (but less than 540°). The perimeter and area of all figures cannot exceed a maximum size.

Analytic approaches. Geometry was almost exclusively synthetic in nature until the 17th century. In 1637, René Descartes introduced analytic geometry. He established what we now call the Cartesian coordinate system (although he only defined the first or positive quadrant). He demonstrated that many hitherto unsolved geometric problems were solvable by means of analytic geometry. Three key ideas separate analytic from synthetic geometry. First, numbers are associated with the locations or coordinates of points. Second, equations are associated with curves. Third, coordinate equations are used to define distance and other metric properties.

Descartes's ideas proved to be extremely important. Algebra and geometry merged into one discipline. As geometry could now be quantitative, mathematicians put more effort into the study of algebra. The calculus became possible. In short, mathematics became far more flexible and powerful. This analytic approach to mathematics and geometry made many of the profound discoveries of Newtonian physics possible.

The analytic approach can be used to describe all of the synthetic geometries we just mentioned. For a time after Lobatchevsky, synthetic and analytic geometry contested for supremacy. In the end, analytic geometry won the battle. In 1854, Riemann introduced an extremely general form of geometry. The synthetic Euclidean, hyperbolic, and spherical geometries were simply special cases of this more general analytic geometry. Where synthetic geometry had only introduced a handful of possible geometries, Riemannian geometry allowed for a potentially infinite variety. In addition, analytic geometries can make use of powerful tools such as algebra and calculus. Since Riemann's time, synthetic geometry gradually faded from the mathematical world. In fact, one of the few places where it still finds adherents is in psychology (as we will see later).

In Riemann's terms, Euclidean, hyperbolic, and spherical geometries are referred to as geometries of constant curvature. A Euclidean geometry is considered flat and has a curvature of zero. A spherical geometry has a constant positive curvature, and a hyperbolic geometry has a constant negative curvature.

Metric spaces. A Riemannian geometry has two essential characteristics (Eisenhart, 1925): First, the space must be a manifold. That is, there must be some way to assign coordinates to the points in the space, and functions assigning these coordinates must be smooth. (There should be no discontinuities between the coordinates of points lying close to each other.) Second, the nature of the space is critically related to the distance function that is defined on the space. Different distance functions are indicative of different spaces. In fact, in 1871

Klein showed that Euclidean, hyperbolic, and spherical geometries essentially only differ in their respective distance functions.

Riemann's ideas are stated in their most general form in modern topology and real analysis. One of the most general types of distance-defined spaces is called a *metric space*. A metric space consists of two parts. First, there must be a non-empty set of points (X). Second, there is a function (d) defined on the set which assigns a distance to any pair of points (x,y). Such a distance function, called a metric, must satisfy four conditions:

Let x, y, and z be elements of set X, then d(x,y) is a metric on X if

(1) Distance is always non-negative. That is,

$$d(x,y) \geq 0. \tag{2.1}$$

(2) Non-identical points have a positive distance. That is,

$$d(x,y) = 0 \text{ if and only if } x = y. \tag{2.2}$$

(3) Distance is symmetric. That is,

$$d(x,y) = d(y,x). \tag{2.3}$$

(4) Distance is the shortest path between points. In other words, a path between two points which is traced through a third point can never be shorter than the distance between the two points. (There are no short cuts.) This property is often called the triangle inequality. That is,

$$d(x,y) \leq d(x,z) + d(z,y). \tag{2.4}$$

These metric axioms express much of what is essential to our every day concept of distance.

The metric axioms are also quite general. An infinite variety of possible metric spaces exist. Three of the most well known metrics are the Euclidean, city block, and Minkowski metrics. Let's look at these three metrics as examples of metric functions.

In a two dimensional Cartesian coordinate system, the Euclidean distance between two points P_1 and P_2 located at the coordinates (x_1,y_1) and (x_2,y_2) respectively is

$$d(P_1,P_2) = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2}. \tag{2.5}$$

This is the typical "straight line" distance between two points. The metric space defined in this way has all the properties of a Euclidean space.

Distance need not be defined in a Euclidean manner. Other metrics may be more natural under various circumstances. For instance, imagine that you are in New York City at 96th street and 1st Avenue, and you want to walk to a diner at 90th street and 3rd Avenue. Unless you can walk through buildings, the distance you would need to walk would not be the Euclidean, as-the-crow-flies distance. In this case a more realistic conception of distance is that your destination is six blocks downtown and two blocks cross-town. In other words, the diner is eight blocks away. The metric we have just described is appropriately called the city block metric. It is expressed mathematically as

$$d(P_1,P_2) = |x_1-x_2| + |y_1-y_2|. \tag{2.6}$$

A final example shows how general and powerful metrics can be. A third type of metric is the Minkowski metric, defined as

$$d(P_1,P_2) = [|x_1-x_2|^R + |y_1-y_2|^R]^{1/R}. \tag{2.7}$$

Here, *R* may take on any positive value greater than or equal to one. If R=1, the city block metric results and we have Equation 2.6. If R=2, the Euclidean metric results and we have Equation 2.5. Clearly, *R* can take on an infinite number of values resulting in a infinite number of potential metric spaces. Metrics also exist for hyperbolic and spherical geometries.

People are capable of looking at distance, of creating metrics, in more than one way, and stimulus conditions can also influence the metric used. As a simple example, most city dwellers tend to think of the distance between places in terms of a driving time metric. Because different roads travel at different speeds at different times of the day, this driving time metric would be quite complicated and very non-Euclidean.

A second more whimsical metric should be familiar to anyone who lives in a cold climate. I might call it the pain metric. People are often willing to walk a bit out of their way as long as they can stay out of the cold. The pain metric, then, would be the path that produces the minimum amount of pain from the cold.

An interesting account of metrics and their applications can be found in Shreider (1974). In terms of the psychology literature reported later in this book, we will see that the metric of visual space under laboratory conditions varies depending on which instructions are given to the subject and stimulus conditions. One of the primary themes of this book is that there is no single metric that describes visual space.

Metrics are sometimes stated in a differential form. Here we assume that if the metric function is true on a large scale it is also true on an infinitesimal scale. In terms of differentials, we would express the Euclidean metric (Equation 2.5) as