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Dynamo theory then and now

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Abstract

A brief history of dynamo theory is presented, from its earliest beginnings, through the development of successful kinematic models (those in which only the electrodynamic equations are solved), up to the present time when fully magnetohydrodynamic simulations have successfully reproduced the main features of the Earth's magnetic field. A particular focus of this paper is the role of the solid inner core of the Earth on the dynamics of its fluid core. Some new results are presented concerning the age and topography of the inner core. © 1998 Elsevier Science Ltd. All rights reserved.

1. The kinematic geodynamo

Although it has been known for many centuries that the Earth is magnetic [1–3], the reason for this, and for many puzzling features of the Earth's magnetic field, have been convincingly explained only during the present century. The key was the discovery in 1906 that the Earth possesses a fluid core [4]. To be sure, the curious time scales of the field, long compared with those of the atmosphere and oceans, but short compared with geological processes, had suggested to people like Halley [5] and Hansteen [6] that fluid motions within the Earth must somehow be involved, but nothing was certain until 1906. The density of the fluid core, deduced from seismological observations, ranges from 9904 kg m^{-3} at the top to 12166 kg m^{-3} at the bottom [7]. This suggests that it is largely composed of molten iron, compressed by the 136–329 GPa hydrostatic pressures created by the weight of the overlying mantle. Iron is a good electrical conductor, and it was not long (1919) before Sir Joseph Larmor [8] proposed that fluid motions in the core create the Earth's magnetism through self–excited dynamo action.

Although today no one seriously doubts that Larmor's idea was correct, the development of a geodynamo model soon encountered difficulties. A man–made dynamo is carefully constructed to create electrical currents as efficiently as possible; the naturally–occuring

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dynamo in the Earth operates in an almost spherical body of nearly homogeneous electrical conductor, where short–circuiting of electrical currents is severe. The very different character of man–made and naturally–occuring dynamos became apparent in 1933, when Cowling [9] published his famous theorem, that a dynamo cannot maintain an axisymmetric magnetic field; (see also [10]). This was a severe blow to theoreticians who had hoped that the predominantly axisymmetric form of the observed geomagnetic field pointed to the probable existence of axisymmetric dynamo–created fields. The search for fluid dynamos became three–dimensional (3D) and, therefore, far more difficult, though there was a pervasive sentiment that perhaps Cowling's theorem was the foretaste of a stronger result that would rule out self–excited fluid dynamos completely, and in apparent confirmation several other 'anti–dynamo theorems' were soon discovered [11, 12].

Optimists, such as Walter Elsasser and Sir Edward Bullard, held staunchly to the quest. They independently argued [13, 14] that, although the energy budget of the core was tight, sources existed, most probably thermal convection or motions induced by the luni–solar precession, that would suffice to power the geodynamo. They focussed research on the so–called 'kinematic dynamo problem', that of finding solutions to the electrodynamic equations and boundary conditions alone. The former consist of the pre–Maxwell equations

$$\partial_t \mathbf{B} = -\nabla \times \mathbf{E} \tag{1}$$

$$\mu_0 \mathbf{J} = \nabla \times \mathbf{B} \tag{2}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{3}$$

and Ohm's law for a moving conductor

$$\mathbf{J} = \sigma(\mathbf{E} + \mathbf{V} \times \mathbf{B}). \tag{4}$$

Here $\partial_t = \partial/\partial_t$, $\mathbf{B}$ is magnetic field, $\mathbf{E}$ is electric field, $\mathbf{J}$ is electric current density, σ is electrical conductivity, and μ_0 is the permeability which, because of the high prevailing temperatures, is taken to be that of free space ($4\pi \times 10^7$ H m^{-1}). The fluid velocity, $\mathbf{V}$, is specified and the question is posed whether (1)–(4) admit solutions that are continuous at the surface of the conductor with a source–free potential field in the surrounding insulator, and which further are self–excited, i.e. do not disappear with increasing time, t. Because the answer is, 'No', for axisymmetric $\mathbf{B}$, the kinematic dynamo problem is a tough nut to crack. Though a linear mathematical problem, it requires that 3D solutions be found to the 'induction equation'

$$\partial_t \mathbf{B} = \nabla \times (\mathbf{V} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}, \tag{5}$$

a vector partial differential equation that follows from (1)–(4). Here $\eta = 1/\mu_0\sigma \approx 1$ m^2 s^{-1} is the 'magnetic diffusivity' of the core. For further detail about the formulation and solution of the kinematic dynamo problem, see for example [15, 16].

A few years after the publication of Bullard's paper [14], one of us (PHR) became a research student at Cambridge University, and could occasionally interact with Bullard, then the director of the National Physical Laboratory (NPL) at Teddington, England. It was clear that Bullard hoped to obtain solutions numerically through the computing resources of NPL, then considered massive, though considerably less capable than a modest workstation is today. Bullard aimed to solve the simple steady–state version of (5). Writing

$$\mathbf{V} = R\mathbf{U}, \tag{6}$$

with a specified $\mathbf{U}$, he hoped to find a real R such that

$$0 = R\nabla \times (\mathbf{U} \times \mathbf{B}) + \eta\nabla^2\mathbf{B} \tag{7}$$

possesses nontrivial solutions. He developed a spectral approach, now known as the Bullard–Gellman formalism [17], in which (5) was transformed into a set of ordinary differential equations for the spherical harmonic components of $\mathbf{B}$.

It was soon after this that one of us (PHR) was privileged to become Professor Chandrasekhar's Research Associate. Chandrasekhar, or "Chandra" as he was generally called, was at that time engaged in solving numerous fluid dynamic stability problems by variational methods, an activity that led to the publication of his celebrated book [18]. He clearly saw (5) as yet another linear stability problem: given a steady $\mathbf{V}$, (5) poses an eigenvalue problem for the growth rate $\lambda = \partial_t$ of the field:

$$\lambda\mathbf{B} = \nabla \times (\mathbf{V} \times \mathbf{B}) + \eta\nabla^2\mathbf{B} \tag{8}$$

Chandra also recognized that Bullard's hope of finding an R for which $\lambda = 0$ might be forlorn. He realized that the eigenvalue problem posed by (8) is not self–adjoint and therefore is unlikely to possess real eigenvalues. He confided to one of us (PHR) that the search should not be for an R, for which λ is zero, but for an R for which $Re(\lambda) = 0$.

Even Chandra, though seldom deterred by technical obstacles, was daunted by the difficulty of solving (8), but he did ask an ingenious question that was potentially easier to answer [19]: though all axisymmetric solutions of (5) must die away, is it possible to reduce $Re(\lambda)$ so greatly that the decay is lengthened enough to make axisymmetric solutions again of geophysical interest? Unfortunately, this idea did not work out well; Chandra's own research student, George Backus, showed [20] that $Re(\lambda)$ could not be reduced by more than a factor of 4 below the reciprocal of the free decay time, $\tau_\eta = R_1^2/\pi^2\eta$, in which fields decay when $\mathbf{V} = 0$. Here R_1 is the radius of the core-mantle boundary (CMB), so that τ_η is only of order 10^4 years, a short time in comparison with the age of the geomagnetic field, which is known from the study of paleomagnetism to be not much less than that of the Earth (4.5×10^9 years). A related idea did, however, work out well. Braginsky [21, 22] showed that, though the axisymmetric dynamo fails, the 'nearly' axisymmetric dynamo succeeds. Chandra was also very interested in what he felt was the likelihood that MHD turbulence can regenerate magnetic field.

Dynamo theory was turned round in 1958 by two independent demonstrations, both essentially using asymptotic methods, of working spherical dynamos, one by Arvid Herzenberg [23, 24] and one by George Backus [25]. Soon afterwards, two simple analytic dynamos were created in cylindrical geometry [26, 27]. All these models were implausibly artificial from the standpoint of geophysics, but eventually computers became powerful enough to generate more realistic kinematic geodynamos, via the Bullard–Gellman spectral method [28–31]. Most models assumed for simplicity that Earth's core is totally fluid, but in reality it has been known since 1936 that a solid core lies at the center of the Earth [32].

2. The magnetohydrodynamic geodynamo

As it became increasingly clear that the kinematic dynamo problem was ‘under control’, growing attention was paid to the magnetohydrodynamic (MHD) dynamo problem, in which both **B** and **V** are sought, that self–consistently satisfy not only (5) and the related boundary conditions but also the equations and boundary conditions of fluid motion. This coupled MHD system is fully nonlinear, and is therefore much harder to solve than the induction equation. Though one ingenious planar model was devised and partially solved by asymptotic methods [33, 34], numerical integration is generally the only way forward. Fortunately, developments in computer hardware and advances in numerical methods have met the greater challenge presented by MHD dynamos.

The question of the energy source must be re-opened: What powers the geodynamo? Since 1963, compositional convection has gradually become the favored mechanism. Analysis of seismological data indicates that the density, ρ, of the fluid core is less than it would be were it predominantly made of iron at the prevailing pressure, P, and temperature, T, and that therefore the fluid iron must be alloyed with lighter elements such as Sulfur, Silicon and Oxygen. Both P and T increase with depth, but it is the increase in P that has the greater effect on the phase of core material. Jacobs [35] suggested that this results in a phase change at the inner core boundary (ICB), the material beneath being the solid inner core (SIC) and that above being the fluid outer core (FOC). Verhoogen [36] observed that, as the Earth loses heat to space and the core cools, the phase boundary (the ICB) must move upwards, releasing the latent heat of solidification at the ICB, which may suffice to drive thermal convection in the FOC. There is a density jump between phases at the ICB of $\Delta\rho \approx 600$ kg m^{-3} [7], which is much larger than would be expected from contraction on solidification alone, and it would, in any case, be very unusual for a fluid alloy to retain the same composition after freezing. A more natural explanation is sketched in Fig. 1, where for simplicity the core is assumed to be a binary alloy, and where the mass fraction, ξ, of light constituent is plotted against the thermodynamic state, as represented (for simplicity, see above) by the pressure, P, alone. At the surface of the ICB the composition of the FOC lies on the liquidus shown on the left, the composition of the SIC then being that of the associated solidus. Solidification results not only in the release of latent heat at the ICB but also in the release of the light constituents, which are possibly more important sources of buoyancy than latent heat, as Braginsky [37] first observed. Even in the absence of the latent heat source, the gravitational energy made available as differentiation at the ICB continually brings the heavy constituents (mainly iron) ever closer to the geocenter is amply sufficient to satisfy the energy requirements of the geodynamo over the entire age of the Earth [38]. A model driven in this way is called a ‘gravitationally–powered geodynamo’.

Because the thermal and compositional releases at each point of the ICB are both proportional to the local rate of advance of the ICB, thermal and compositional buoyancy act in concert. Until recently, all MHD geodynamo models lumped the two sources of buoyancy together in a simple ‘temperature’; more precisely, if they have considered ξ at all, they have regarded the real temperature, T, as a surrogate for it. They have also always ignored the evolution of the Earth, supposing that the buoyancy sources act steadily. They have adopted the Boussinesq approximation, in which all properties of the fluid are assumed uniform, except

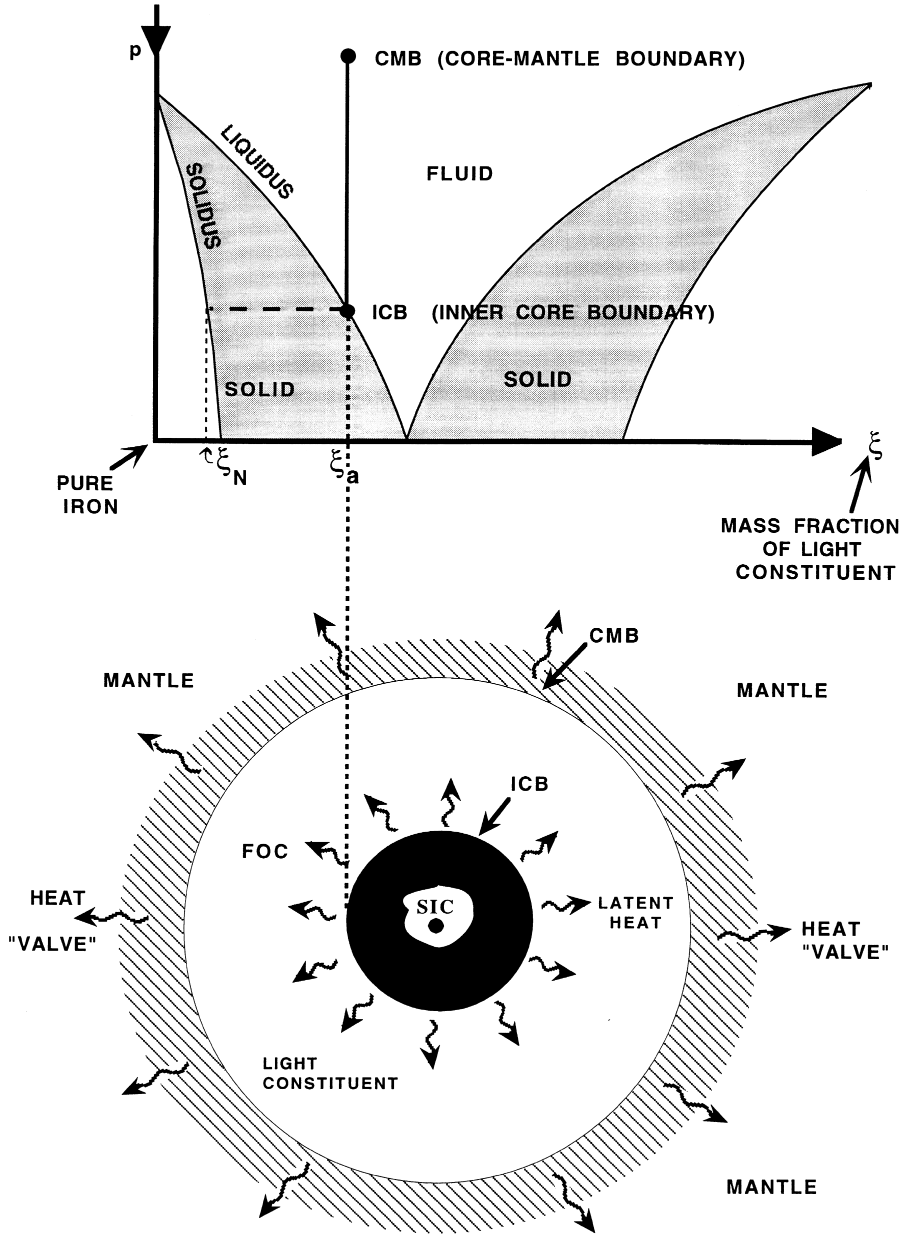

Fig. 1. Illustration of the basic idea of the gravitationally powered geodynamo. Conditions of phase equilibrium are met at the boundary of the inner core, as indicated by the (ξ, P) plot at the bottom of the figure.

that the small changes in ρ created by T are included in the buoyancy force. The resulting Boussinesq equations are then

$$\nabla \cdot \mathbf{V} = 0, \tag{9}$$

$$\partial_t \mathbf{V} + \mathbf{V} \cdot \nabla \mathbf{V} + 2\Omega \times \mathbf{V} = -\nabla \Pi - \alpha T g + \rho^{-1} \mathbf{J} \times \mathbf{B} + v \nabla^2 \mathbf{V}, \tag{10}$$

$$\partial_t T + \mathbf{V} \cdot \nabla T = \kappa \nabla^2 T + \epsilon. \tag{11}$$

Here v is the kinematic viscosity, κ is the thermal diffusivity, α is the coefficient of thermal expansion, $\boldsymbol{g}$ is the acceleration due to gravity, ϵ represents volumetric heat sources (if any), and Ω is the angular velocity of the reference frame (a constant, representative of the Earth's mantle), $2\Omega \times \mathbf{V}$ being the Coriolis acceleration, the associated centrifugal acceleration being included in the reduced pressure $\Pi = (P/\rho) + 1/2\Omega^2 r^2$, where $r = |\mathbf{x}|$ and $\mathbf{x}$ is vector distance from the geocenter.

In 1952 Chandra [39] published a paper on Boussinesq convection in a self-gravitating ($\boldsymbol{g} \propto \mathbf{x}$) sphere containing a uniform distribution of heat sources (ϵ = constant). His solution was elaborated by Backus [40] and Roberts [41]. Chandra [42] also initiated studies of convection in a spherical annulus and of the effect of rotation on convection in a sphere [43]. His student Bisshopp developed the latter topic [44], followed by Roberts [45, 46], Bisshopp and Niiler [47], Busse [48] and Durney [49, 50]. All these were marginal convection studies. A systematic attack on the nonlinear problem was undertaken by Zhang and Busse [51–54] with the special aim of finding a convectively–driven dynamo, in which they were successful. Before, however, they could approach geophysically realistic parameter values, they experienced insuperable truncation problems. They were also compelled to restrict themselves to solutions having an oversimple time dependence, namely those that are steady in a frame rotating longitudinally relative to the mantle. For these reasons, we shall consider here only our own integrations [55–57] that, with one exception, are geophysically unexceptionable in their choice of parameter values, and which do not prejudge the time–dependence of the solution.

The one exception is the kinematic viscosity, v, which from numerical necessity was too large to be geophysically plausible. Although v for the core is very uncertain, most estimates of the molecular viscosity do not exceed 10^{-6} m^2 s^{-1}. Arguments that momentum transport is enhanced by core turbulence generally estimate that a turbulent v will not exceed 1 m^2 s^{-1}. The resulting Ekman number, $E = v/2\Omega R_1^2$, the appropriate dimensionless measure of core viscosity, is less than 10^{-9}, which is still numerically unattainable. We were compelled to assume that $E = 10^{-6}$, which means that v exceeds the most optimistic geophysical value by three orders of magnitude. Whether E is 10^{-6} or 10^{-9}, it is reasonable to suppose that the effects of viscosity are small except in boundary layers at the ICB and CMB (and except possibly in shear layers surrounding the tangent cylinder; see Section 3). In other words, our unrealistic v does not necessarily detract from the applicability of our simulations to the Earth.

Our model was by no means the first MHD dynamo. In addition to early models of Busse [58, 59], Chandra's analysis of the rotating Bénard layer [60, 18] was extended by Childress and Soward [32] and Soward [33] to small, but finite, amplitude motions; they showed analytically that these could maintain weak magnetic fields. Subsequent numerical work by Fautrelle and Childress [61] and St Pierre [62] proved that they could also maintain a strong field. Nordlund *et al.* [63] constructed a further planar model. Gilman and Miller [64], Gilman [65], Glatzmaier [66–68] and Kageyama *et al.* [69] integrated models in spherical geometry. Although these were all converged numerical solutions, none approached geophysically realistic parameter values; indeed in most cases they were intended to simulate the solar dynamo and not the geodynamo. Not surprisingly, none of the models produced Earthlike magnetic fields.

The magnetic field produced by our Boussinesq dynamo possesses several features resembling the observed geomagnetic field. Most remarkably, during the 50000 yr of simulated time over which it was studied, the model underwent a polarity reversal similar to those that the geomagnetic field has experienced many times during the Earth's history [70]. There were some other surprises, notably the important role played by the SIC in the dynamics of the FOC, a feature shared by all our later simulations; see Section 3 below.

Next, we broke completely new ground [71, 72] by studying inhomogeneous models of the core, ones that allow for the increase of ρ with depth, that include both thermal and compositional buoyancy distinctly, and in which these are solely responsible for driving the geodynamo, there being no other energy sources, such as radioactivity. In other words, the models are evolutionary. The theory on which these models are based had previously been developed in great detail by Braginsky and Roberts [73]. We may fairly claim that, with the exception of ν, our models do not contradict any firmly established geophysical fact that is relevant to core MHD and the geodynamo. And the magnitude and harmonic structure of the magnetic field that the models create in fact resemble those of the present day geomagnetic field, i.e. they are characteristic of the field between reversals.

3. The inner core

Most early kinematic and MHD geodynamo simulations either left out the inner core entirely, filling the vacant space with core fluid, or they included it not so much for geophysical realism but in order to avoid numerical complications arising from the coordinate singularity at the geocenter, $r = 0$. Bearing in mind that the SIC contains only 5% of the mass of the core and 4% of its volume, its neglect appeared as a matter of little significance. Also, if included, it did not seem to matter much whether it was modeled by an insulator or a perfect conductor (the simplest cases) rather than by a material of similar properties to the FOC. All that has changed dramatically during the last 3 years.

It was early recognized that, being to free to turn in response to the torques to which the FOC subjects it, the SIC must rotate at a slightly different rate from the mantle. The first kinematic models that included an inner core [74] predicted relative motions of a little less than 0.1°/yr, westward or eastward relative to the mantle, depending on the model. Arguments were presented [75] that favored westward motion. In contrast, our simulations [55–57, 71, 72] were unanimous in predicting variable eastward motion of typically 1–3°/yr. These predictions have recently been corroborated by an analyses of the seismic data by Song and Richards [76], later corroborated by Su *et al.* [77].

We have recently [72] uncovered the dynamical reasons for the direction and magnitude of inner core rotation. In summary, Coriolis forces tend to allow convection to carry heat away from the rotation axis more efficiently than parallel to it. In fact, apart from boundary layers, the FOC outside the tangent cylinder is almost isentropic. The tangent cylinder is an imaginary surface that touches the ICB at its equator and is parallel to Ω. Heat transport within the tangent cylinder is inhibited by rotation, and a temperature difference between ICB and CMB large enough to overcome this obstacle has to develop. As a result, the ICB tends to be hotter near its poles than at its equator. For the same reason, the convective circulations between the ICB and CMB are upward near the rotation axis, the return downward flow being further from the axis. Coriolis forces deflect these motions westward near the CMB and eastward near the ICB. In broad agreement with Le Chatelier's Principle, the SIC tends to respond by moving in the same eastward dircetion as the fluid lying just above it. The main coupling that brings this about is electromagnetic. The SIC acts in a similar way to the armature of a synchronous motor, corotating with the fields threading it to the eastward moving fluid immediately above it, and powered by the electric currents generated by the dynamo operating in the FOC.

It has been suggested [77] that the viscous torque, Γ_ν, on the ICB is also essential, but to show that this is not so we repeated our integrations with stress–free conditions replacing no–slip conditions on the ICB [72], and we found that the relative motion is unchanged in direction and scarcely altered in magnitude. To understand this, suppose first that the moment of inertia, I_C, of the SIC is zero and that $\nu = 0$. Then the SIC will continuously adjust its angular velocity, Ω_{IC}, to make the electromagnetic torque, Γ_B , zero. If $\nu \neq 0$ (still with $I_C = 0$), Ω_{IC} will adjust continuously to make $\Gamma_B + \Gamma_\nu = 0$. In this sense, Γ_B and Γ_ν always have the same magnitude, as indeed we found in our numerical integrations. The situation is not substantially changed when ν is nonzero, but small. In particular, the fields on the ICB creating Γ_B are scarcely altered. Paradoxically, because the magnetic stresses are so much larger than the viscous stresses, they must almost cancel when integrated over the ICB to create Γ_B. Of course, I_C is, in reality, nonzero so that a phase lag exists between the cause, Γ_B, and the effect, Ω_{IC}, but this is so short that $I_C = 0$ is a good approximation to geophysical reality, and this was assumed in our original simulations [55–57].

Even in the simplest non–magnetic non–convecting (non-Earthlike) case where E is small, a relative motion between the SIC and mantle establishes circulations in the FOC which are driven by Ekman layers on the ICB and CMB, one of which draws in fluid from the bulk of the FOC, the other pumping it out. These circulations are confined to the interior of the tangent cylinder. The spatial gradients of the flow are large in shear layers surrounding the tangent cylinder, across which the flow has to adjust from the circulations inside the cylinder to stagnant conditions outside, stagnant because our reference frame moves with the mantle. The solution of this complicated hydrodynamic problem was first provided by Stewartson [78]. The effect on Stewartson's solution of a meridional field having lines of force that cross the tangent cylinder has been studied by Hollerbach [79] and Kleeorin *et al.* [80].

Our simulations have all shown that, despite the anticipated insignificance of the SIC, it and its attendant tangent cylinder are decisive in determining the structure of the MHD convection and the resulting dynamo. Hollerbach and Jones [81, 82] had earlier pointed out that the SIC has an electromagnetic time constant, $\tau_{IC} = \bar{R}^2/\pi^2\eta$, exceeding 10^3 years, which is somewhat larger than typical overturning times of the FOC, which are at most a few hundred years. (Here $\bar{R}$ is the mean radius of the SIC.) They argued that, since the field threading the SIC would have to invert itself during a polarity reversal at much the same time as the field everywhere else, the SIC would tend to inhibit such reversals. Our simulations have fulfilled their expectations.

Our most recent simulations [71, 72] have all been integrations of the evolutionary model described in Section 2, in which buoyancy is provided at the ICB as heat (described by an entropy perturbation, S) and as composition (described by the perturbed mass fraction of light constituent, ξ). These sources are proportional to each other and to the rate at which the ICB moves upwards through freezing. They are not however uniform over the ICB. Where the cold convection currents created by the sources descend onto the ICB, the rate of freezing is enhanced; where the hot rising flows are initiated, the rate of advance of the ICB is slowed. For these reasons, the radius $R(\theta, \phi, t)$ of the ICB is a function of colatitude θ and longitude ϕ, and the ICB therefore has 'topography', $h = R\bar{R}$.

Variations in rates of freezing do not produce the only or the largest topographies of the ICB. First and foremost, since the inner core rotates at roughly the same angular velocity as the Earth as a whole, it is flattened by centrifugal force. The resulting equatorial bulge has been estimated to be about 3 km [83]. Second, as Buffett [84] has recently observed, the inhomogeneities in mass distribution in the mantle distort the surfaces of constant gravitational potential within the core. As a result, the conditions for phase equilibrium that define the surface of the inner core are met at varying distances from the geocenter. He estimates that for this reason the otherwise spheroidal shape of the inner core surface could be distorted by as much as 100 m. In what follows we shall ignore these two effects, and focus on the topography created by the convection currents, which we regard as being superimposed on the those produced by the other two mechanisms.

Consider first the evolution of $\bar{R}(t)$, the spherical average of R. The net heat flux from the core to the mantle is not well known. Simulations of mantle convection [85] suggest that the value we adopted (7.2 TW) is a reasonable guestimate. This exceeds the flux of heat down the adiabatic gradient, which is about 5 TW. The difference of 2.2 TW comes from the convective heat flux just below the CMB. We suppose that the entire heat flux emerges uniformly over the CMB. We find that $\bar{R}$ then increases at an average rate of

$$\dot{\bar{R}} \approx 10^{-11}\, m\, s^{-1} \approx 3\,\mathrm{cm/century}. \tag{12}$$

The relative change in $\bar{R}$ over the 70000 years spanned by our anelastic simulation is only about 10^{-5}; so we did not feel it necessary to adjust $\bar{R}$ in our integrations. If we assume the volume of the SIC has always grown at the rate (12), the age of the inner core is

$$\tau_{\mathrm{IC}} = \frac{\bar{R}}{3\,\dot{\bar{R}}} \approx 10^9\, yr, \tag{13}$$

which is less than a quarter of the age of the Earth. This may, however, be too simplistic. The actual age of the SIC would be determined by the thermal history of the Earth, especially the the time-dependent heat flow out of the core, though according to [86] this has not varied greatly during the SIC's existence. More significantly, the presence of radioactivity in the core could create equally vigorous convection without requiring such a rapid growth of the SIC. Perhaps, therefore, (13) should be regarded as a lower bound. Our estimate of τ_{IC} also relies on facts about the composition and material properties of the core (such as the latent heat of melting, which we took to be 10^6 J kg^{-1}) that are very uncertain. Two other recent estimates of

τ_{IC} derived from global energy balance models may be noted: 2.8×10^9 years (Buffett *et al.* [87]) and 1.7×10^9 years (Labrosse *et al.* [88]).

Consider now the evolution of the topography, h. This depends crucially on the time-dependent convective planform at the ICB, especially the horizontal variation of specific entropy (S). The actual conditions we apply at the ICB are [71, 72]

$$\frac{\partial R}{\partial t} = -c_\mathrm{a}\frac{\partial S}{\partial r} = -c_\mathrm{b}\frac{\partial \xi}{\partial r} = -c_\mathrm{c}\frac{\partial S}{\partial t} - c_\mathrm{d}\frac{\partial \xi}{\partial t}. \tag{14}$$

The basic state and material properties used to construct the positive constants c_a, c_b, c_c and c_d. are described in [71, 73]. The local topography at time t is then

$$h(\theta, \phi, t) = -c_\mathrm{c}S(\theta, \phi, t) - c_\mathrm{d}\xi(\theta, \phi, t). \tag{15}$$

In our simulation the non-axisymmetric part of the radial gradient of the entropy (which determines the non-axisymmetric fluxes of latent heat and light constituent at the ICB and the time rate of change of its topography) is dominated by the longitudinal wave number 2. Fig. 2(a) shows a snapshot of our $\partial h/\partial t$ as a function of θ and ϕ. This pattern drifts over the ICB at a typical phase speed of $0.3°$/yr, westward relative to the mantle, i.e. at about $3°$/yr relative to the SIC. Consequently, a place on the ICB where the surface elevation is growing comparatively rapidly (i.e. where the local fluxes of latent heat and light constituent are relatively large) changes in about 30 yr to a slowly growing area as the fluxes decrease there. Correspondingly, a local high point in the surface elevation (low temperature) changes to a local low point (high temperature) in the same time interval. As a result, the non-axisymmetric growth rate [which is comparable with (12)] never has time to accumulate non-axisymmetric topography greater than about a centimeter.

The situation is quite different for the axisymmetric part of h, which is not 'washed out' by the westward phase velocity of the convection pattern. As noted in our explanation of the inner core rotation, entropy on a spherical surface near the inner core boundary is greater in the polar regions than in the equatorial region. The freezing condition therefore requires the local topography, h, be less at the poles than at the equator. In our simulation the resulting equator–pole difference in elevation is usually about 14 m. This is illustrated in a snapshot of h shown in Fig. 2(b), which also brings out the high degree of axisymmetry of the inner core topography.

Although our geodynamo simulations provide plausible explanations for observations like the inner core super rotation and make predictions about inner core growth, they raise further questions that merit investigation. Our models have assumed phase equilibrium on the inner core boundary, i.e. that, as thermodynamic conditions change, freezing or melting occurs instantaneously to maintain the boundary at the freezing point. No allowance has been made for a finite time of relaxation to such a state. If that relaxation time were long compared with the time scales of interest in our model, the inner core would behave as a solid. As Buffett [84] has noted, the orientation of the inner core would then plausibly be gravitationally 'locked' to that of the mantle by the inner core topography created by mantle inhomogeneities. The relative motion of core and mantle, which we predicted [55] and for which observational evidence has been adduced [76, 77], would then probably not take place. At the opposite

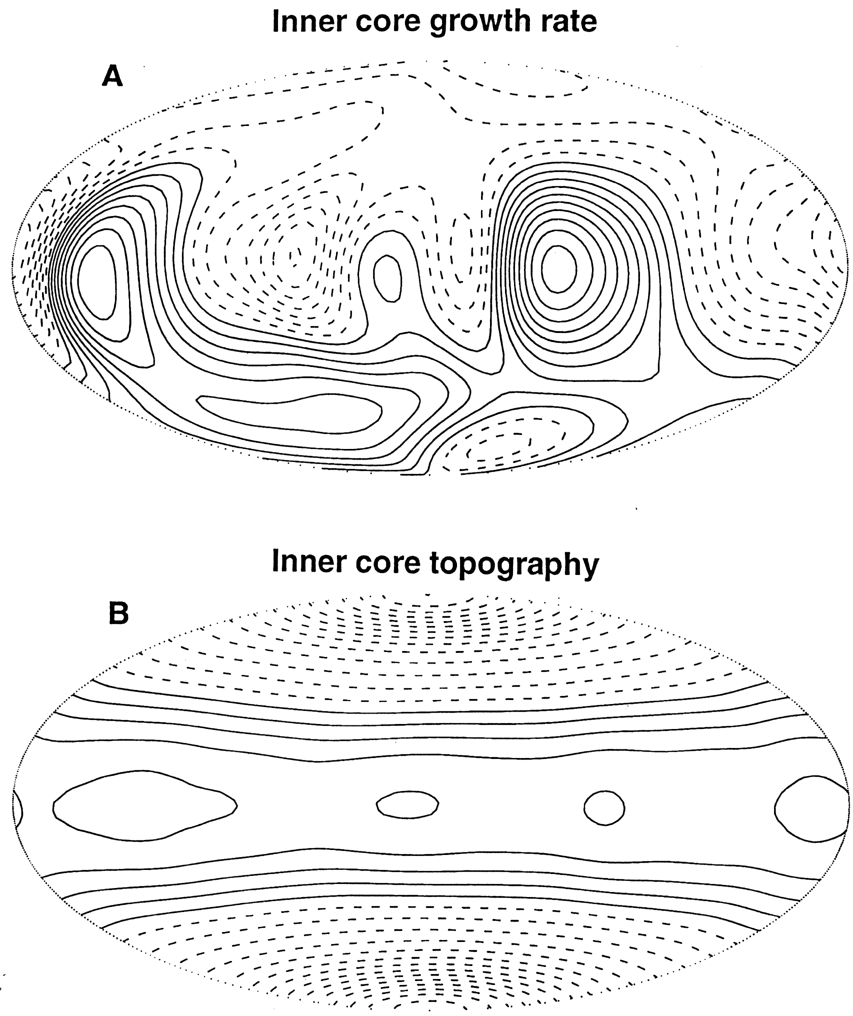

Fig. 2. (a) A snapshot of the local growth rate of the inner core, $\partial h/\partial t$. Solid (dashed) contours represent faster (slower) growth rates; the maximum value is 1.4×10^{-11} m s^{-1}. (b) A snapshot of inner core topography, h. Solid (dashed) contours represent greater (lesser) radius. The radius at the equator is about 14 m greater than that at the poles. Equal area projections are used with the North (South) pole at the top (bottom) of the images.

extreme, in which the melting–freezing relaxation time is short compared with time scales of interest, the topography on the ICB created by mantle inhomogeneities would not lock the inner core to the mantle, since that topography would, by melting in one area and freezing in another, reform as fast as the inner core rotates relative to the mantle. Presumably the truth lies somewhere between these two extremes, a question that it remains for the future decide.

The answer may lie in a curious fact: although we speak of a solid inner core, there are compelling reasons for believing [89] that the top of the SIC is a mixed phase region of the type that metallurgists often call a 'mushy zone'. The mushy layer would not be uniform in thickness, since rising flow tends to melt mush, while descending flow promotes its formation.

The result would be further enhancement of the equator–pole difference in surface elevation. Loper [90] suggests that the organization of the flow within the mushy layer by the overlying meridional flow may give the mush a preferred 'fabric', that is preserved as the mush descends relative to the ICB and solidifies. He speculates that this might explain the observed seismic anisotropy of the SIC [91, 92]. We may also expect that the existence of a mushy zone would promote conditions of local thermodynamic equilibrium so inhibiting gravitational locking of the SIC to the mantle. It is clear that geodynamo theory has advanced far since the days when one of us (PHR) was indelibly and beneficially affected by Chandra through discussion on these and similar fascinating topics.

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